

# The Cross-Link Tecnique for Deep Space Missions

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**Abstract**— Availability of a precise time reference is a key issue for several space missions. Although atomic clocks already have a good flight experience, they also have heavy requirements in terms of volume, mass, power and cost, which do not fit all applications. An alternative to this solution is the use of a good precision oscillator which shall be synchronized to a ground station master clock. The proposed paper illustrates the findings of a recent study promoted by ESA on in-space synchronization methods. The selected approach is based on the cross-link technique.

## I. INTRODUCTION

The classical crosslink technique [1][2][3], associated with an event count carried out at both the GS (Ground Station) and the SC (Spacecraft), allows for a preliminary estimate of the Doppler which can be usefully exploited to implement a frequency and time transfer between the GS and the SC.

Actual asymmetries in the link paths will clearly involve errors on the frequency and time transfer processes. Specifically, performance depends on several error sources, namely geometry differences between uplink and downlink, troposphere and ionosphere propagation effects, solar plasma. Also, a host of relativistic effects (namely Sagnac, red shift, second order Doppler, Shapiro) affect the solution.

Some of these error sources can be analytically investigated, while for others a simulation is the best way to provide an estimate of their effect. Moreover, since the Shapiro delay is a third order effect (it depend on  $c^{-3}$ ), it can be considered negligible compared with the other second order relativistic effects at the accuracy levels achievable by the analysed technique for deep space missions.

In order to obtain a figure for the performance of the crosslink technique for deep space missions, a purposely developed code has been implemented under MATLAB environment. By including the specific mission profile, it is

possible to estimate the accuracy of the time and frequency transfer depending on the selected trajectory.

The specific case for a mission to Jupiter is considered in the proposed paper, with an evaluation of the synchronization performance.

## II. METHOD DESCRIPTION

Fig.1 shows a diagram which describes the procedure used by the adopted method. The Doppler tracking is achieved by sending data frames of known duration  $T_U$  from the GS to the SC and by measuring its length, at both the GS and the SC, in terms of (fractional) numbers  $N_{EU}$  and  $N_{SU}$  of their own local clock periods. If  $f_E$  and  $f_S$  are the frequencies used at the GS and at the SC respectively and  $T_{UP}$  is the uplink propagation time, the two measures are (see Fig.1):

$$N_{EU} = T_U f_E \quad N_{SU} = T_U (1 + dT_{UP}) f_S \quad (1)$$

where the measure carried out at the SC is affected by the one way Doppler effect described by the factor  $(1 + dT_{UP})$ . Data frames of duration  $T_D$  are also sent from the SC to the GS and the same process is repeated achieving the further values (see Fig.1):

$$N_{SD} = T_D f_S \quad N_{ED} = T_D (1 + dT_{DW}) f_E \quad (2)$$

being  $T_{DW}$  the downlink propagation time. During the feedback transmission the SC also sends to the GS the measures  $N_{SU}$  and  $N_{SD}$  so the GS can finally determine, by using the measured quantities  $N_{ED}$ ,  $N_{SU}$ ,  $N_{SD}$ ,  $N_{EU}$ , the factor:

$$(1 + dT_{DW})(1 + dT_{UP}) = \frac{N_{ED} N_{SU}}{N_{SD} N_{EU}} := \alpha \quad (3)$$

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This study has been financed by ESA GSP (General Study Program). The main role of GSP is to act as “think tank”, laying the groundwork for the Agency’s future activities.

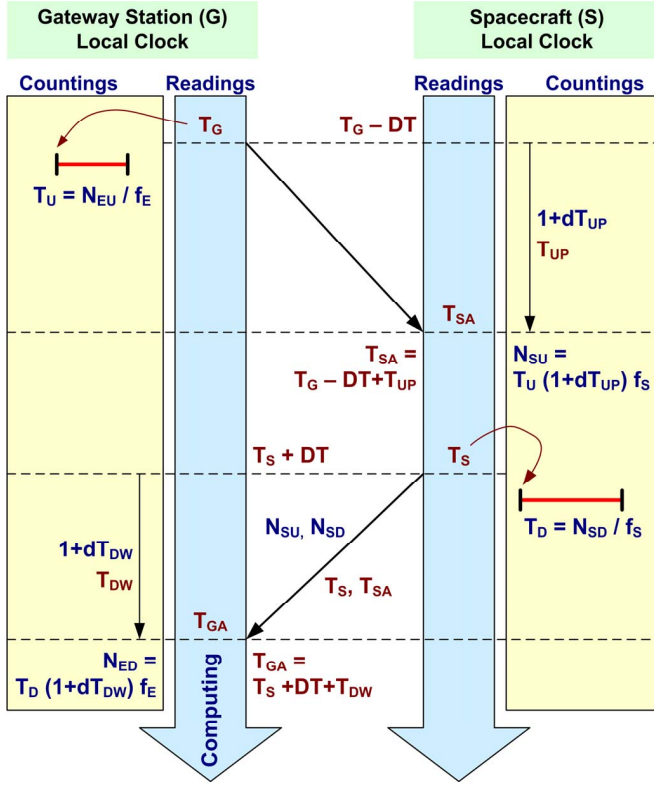


Figure 1. Cross-Link Frequency-Time Transfer with Doppler Estimation

Defining  $\delta$  as the Doppler shift factor  $\delta = \Delta f / f$ , the quantity  $\alpha$  is related to the mean two-ways Doppler shift  $\delta_{2WAY}$  through the following relationship<sup>(1)</sup>:

$$\delta_{2WAY} := \frac{\delta_{DW} + \delta_{UP}}{2} \cong \frac{\alpha - 1}{2} \quad (4)$$

From  $\alpha$ , it is then possible to infer the relative radial speed of the spacecraft:

$$v = c \delta_{2WAY}^{geometric} = c \delta_{2WAY}^{measured} - c \delta_{2WAY}^{errors} = c \frac{\alpha - 1}{2} + \varepsilon_v \quad (5)$$

being  $\varepsilon_v$  the error due to the several effects involved in the measure (e.g. on-board and on-ground oscillators frequency mismatch, atmospheric, relativistic). This speed can be usefully exploited to evaluate the contribution of the second-order Doppler to the accuracy of the time transfer.

Moreover from exploiting the same equations (2), also the frequency transfer can be achieved (i.e.  $f_S$  as a function of  $f_E$ ):

<sup>(1)</sup> The stretching or the shrinking of the transmitted period  $T$  of a relative factor  $dT$  can be associated to a Doppler shift  $\delta = (f_0 - f_1) / f_0$  of a carrier frequency  $f_0$  into a carrier frequency  $f_1$  taking into account that the number of periods  $f_0 T$  in the time interval  $T$  must be equal to the number of periods  $f_1 T(1 + dT)$  after the stretching or the shrinking:  $f_0 T = f_1 T(1 + dT) \rightarrow dT = \delta / (1 - \delta) \approx \delta$ .

$$f_S = \frac{N_{SD}}{N_{ED}} (1 + dT_{DW}) f_E \cong \frac{N_{SD}}{N_{ED}} \sqrt{\alpha} f_E := f_S^{estim} \quad (6)$$

The approximation done in using the estimation defined above depends on the Doppler asymmetry existing between the up and down links:

$$\frac{f_S^{estim}}{f_S} = \sqrt{\frac{1 + dT_{UP}}{1 + dT_{DW}}} \cong 1 + \frac{\delta_{UP} - \delta_{DW}}{2} \quad (7)$$

The quantities  $\delta_{DW}$  and  $\delta_{UP}$  can be estimated by simulation with a residual error  $\varepsilon_\delta$ :

$$\frac{f_S^{estim} - f_S}{f_S} = \left( \frac{\delta_{UP} - \delta_{DW}}{2} \right)_{estim} + \varepsilon_\delta \quad (8)$$

The error  $\varepsilon_\delta$  influences the accuracy of the time transfer because it creates a time error  $\varepsilon_{\Delta t}$  equal to the RTT (Round Trip Time) multiplied by  $\varepsilon_\delta$ . In fact, since  $\delta = v/c$ :

$$\varepsilon_{\Delta t} = \frac{\varepsilon_d}{c} = \frac{RTT}{2} \frac{\varepsilon_{speed}}{c} = \frac{RTT}{2} \varepsilon_{v/c} = \frac{RTT}{2} \varepsilon_\delta \quad (9)$$

Being  $\varepsilon_d$  and  $\varepsilon_{speed}$  the errors in terms of distance and speed corresponding to the Doppler error  $\varepsilon_\delta$ .

The main aim of the crosslink technique is however to compute the difference  $DT$  existing between the GS clock and the SC clock. The proposed approach consists on measuring the time tags  $T_G$  and  $T_{GA}$  at the GS and on sending the time tags  $T_S$  and  $T_{SA}$ , measured at the SC, from the SC to the GS (see Fig.1). The quantities  $R_G = T_{GA} - T_G$  and  $R_S = T_{SA} - T_S$ , then provide:

$$\begin{aligned} R_G &= T_{GA} - T_G = T_S + DT + T_{DW} - T_G \\ R_S &= T_{SA} - T_S = T_G - DT + T_{UP} - T_S \end{aligned} \quad (10)$$

So, finally:

$$DT = \frac{R_G - R_S}{2} - (T_S - T_G) - \frac{T_{DW}^{estim} - T_{UP}^{estim}}{2} + \varepsilon_T \quad (11)$$

The quantity  $(T_{DW} - T_{UP})$  represents the asymmetry existing between the up and down links due to several disturbs and it must be estimated by simulation (with a residual error  $\varepsilon_T$ ) to achieve a sufficiently accurate value of  $DT$ . Finally the two-ways range can also be computed just summing  $R_G$  and  $R_S$ :

$$T_{2WAY} = T_{DW} + T_{UP} = R_G + R_S \quad (12)$$

The disturbs to take into account are mainly: (i) the geometric difference between the up and down link paths, (ii) the atmosphere (troposphere and ionosphere) [4][5] (iii) the solar plasma and (iv) some relativistic effects (red shift, second order Doppler, Sagnac) [6][7]. The residual errors  $\varepsilon_T$  and  $\varepsilon_\delta$  are here considered as standard deviations of random variables.

### III. THE METHOD APPLIED TO A DEEP SPACE SCENARIO

The scenario taken into account in this paper considers a spacecraft orbiting around Jupiter, in a circular orbit at a distance of  $7 \cdot 10^5$  Km from the surface, inside the sphere of influence of the planet (i.e.  $48 \cdot 10^6$  Km). The GS is located at Cebreros (it belongs to the ESTRACK, the ESA Ground Stations Network) and the link is supposed having a carrier frequency of 8 GHz (X band). The results concerning the asymmetries between the up and the down links, achieved by simulation, are summarised in Table I.

It is evident that (i) the geometric effect gives by far the greater contribution to the total asymmetry; (ii) the effects due to the general theory of relativity (red shift, second order Doppler and Sagnac) give contributions to the overall delay asymmetry greater than those due to the atmosphere and to the solar plasma. These disturbs must be compensated to achieve the best time and frequency transfer accuracies.

The compensation leaves residual errors (i.e.  $\varepsilon_T$ ,  $\varepsilon_\delta$ ) whose values depend on several factors: (i) the TEC (Total Electron Content) which is responsible of both the ionosphere and solar plasma effects; (ii) the ZTD (Zenith Total Delay) used to measure the troposphere delay; (iii) the position and the radial velocity of the SC required to evaluate the geometric and relativistic effects. The accuracies considered are (i) 5 TECU (TEC Units, 1TECU= $10^{16}$ el/m<sup>2</sup>) for the TEC and (ii) 15cm for the ZTD; both typically available from the GS facilities (e.g through GPS based measurements); (iii)  $10\mu\text{rad}^{(2)}$  for the SC position and 1m/sec for SC speed.

The results achieved by simulation, using as input these coarse values for the position and speed accuracies of the SC, are reported in Fig.2 and Fig.3.

TABLE I. MAXIMUM VALUES OF THE EFFECTS

Asymmetry	Delay (sec)	Doppler
Geometry	$5.515 \cdot 10^{-1}$	$3.066 \cdot 10^{-6}$
Troposphere	$9.523 \cdot 10^{-9}$	$3.027 \cdot 10^{-12}$
Solar Plasma	$1.108 \cdot 10^{-10}$	$2.403 \cdot 10^{-16}$
Ionosphere	$1.589 \cdot 10^{-9}$	$3.661 \cdot 10^{-13}$
Red Shift	$4.868 \cdot 10^{-8}$	$5.997 \cdot 10^{-15}$
2° Order Doppler	$7.071 \cdot 10^{-7}$	$3.280 \cdot 10^{-11}$
Sagnac	$2.823 \cdot 10^{-7}$	0

<sup>(2)</sup> This accuracy is far from 10nrad state of the art accuracy available by VLBI (Very Long Baseline Interferometry) but it is comparable with 6milli-degrees maximum pointing error available at Cebreros Station corresponding to a standard deviation of  $60\mu\text{rad}$ . 100 independent measures are enough to improve this standard deviation of a factor 10 achieving  $6\mu\text{rad}$ .

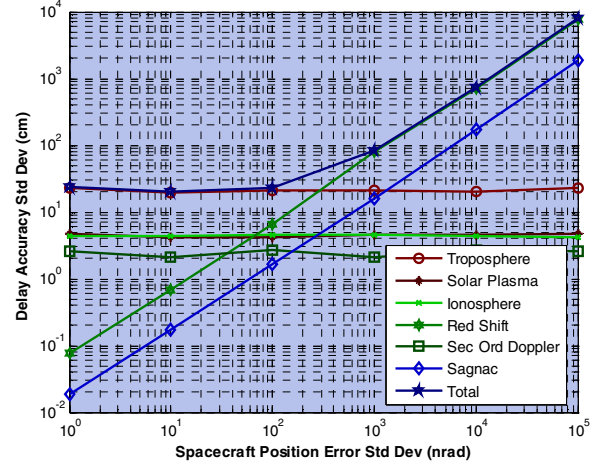


Figure 2. Two-Way Ranging Estimation Accuracy

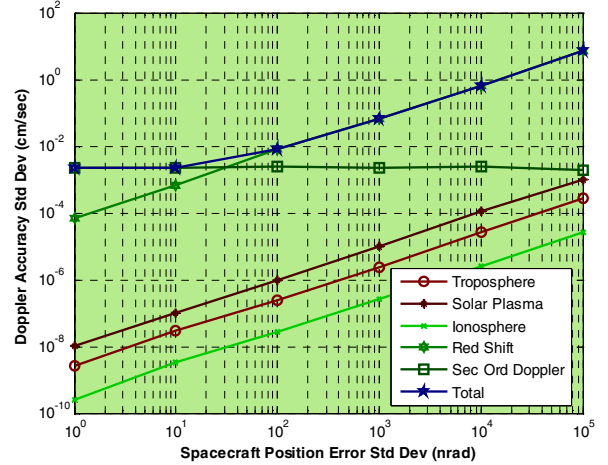


Figure 3. Two-Way Velocity Estimation Accuracy

The values of the curves associated to the labels “Total”, shown in Fig.2 and Fig.3 corresponding to  $10\mu\text{rad}$ , provide the accuracies achievable after compensation for the two way range and the two way speed: about 10m and 1cm/sec respectively.

The two way range and speed can be employed to estimate again, with higher precision, the SC position and speed as shown in Fig.4. This can be obtained through post processing radiometric data gathered by more than one Earth station for enough time arc [8].

Moreover, since the SC position and speed are only available at discrete time intervals, depending on the frame rate used in communicating with the SC, further post processing is also necessary to interpolate them in order to extract the data corresponding to the wanted time instants (e.g. the SC re-transmission point).

Finally also extrapolation of the SC position and speed is necessary to maintain the required accuracy of the SC position after the not-visibility periods or in directions where the multiple observations don't create enough accuracy.

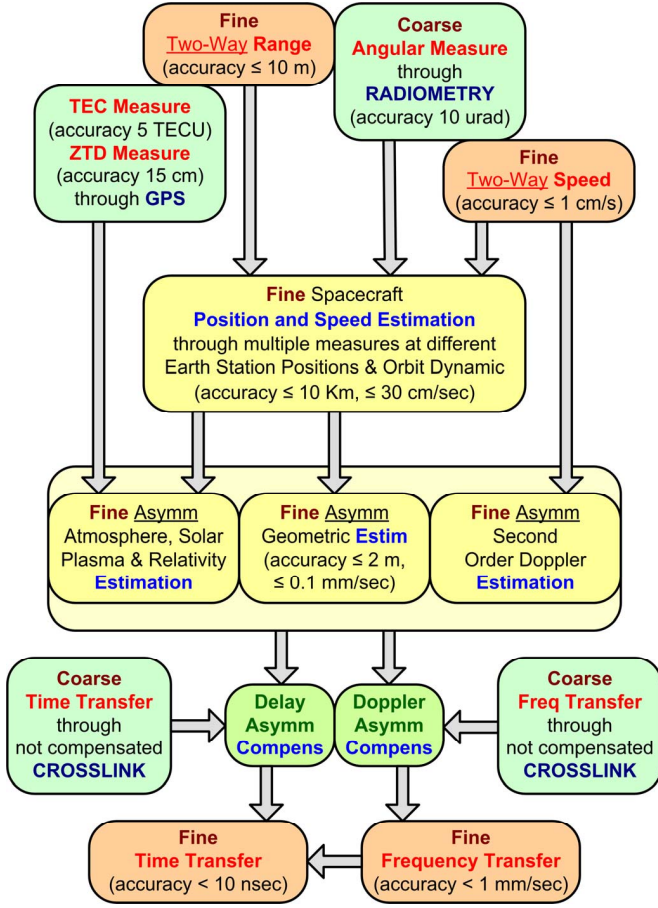


Figure 4. Time Transfer Procedure

The procedure described makes use of extensive post processing as far as the SC position and speed estimations are concerned whereas it provides a straightforward processing of the computation required for time and frequency transfers.

It can easily be proven, by straightforward geometric considerations, that a SC position accuracy of 10Km and a SC velocity accuracy of 30cm/sec are enough to binds the error carried out in compensating the geometric asymmetry to stay within 2m and 0.1mm/sec, for range and speed respectively, under the constraint that there is a single geometric common point for the SC receiving and transmitting time instant. The existence of such a geometric common point is assured by implementing two independent GS-SC and SC-GS asynchronous continuous transmissions of consecutive frames having durations  $T_U$  and  $T_D$  respectively. This asynchronous approach provides in general  $0 \neq T_S - T_{SA} < T_D$ ; however the time gap  $T_S - T_{SA}$  can be made null by correcting the true arrival time  $T_{GA}$  of the feedback signal through the interpolation formula:

$$T'_{GA} = T_{GA} + \frac{T_S - T_{SA}}{T_D} \frac{N_{ED}}{f_E} \quad (13)$$

where  $N_{ED}/f_E$  is the duration of the time interval, sent by the SC, measured at the GS after its reception.

The considered accuracy for the SC position corresponds to about 20nrad when referred to the Earth-Jupiter distance and it allows computing precise values for the other asymmetries (see Fig.5 and Fig.6): 800ps and 0.01mm/sec for the time and speed accuracy respectively. The so obtained values for the asymmetries can be used to compensate the coarse measures obtained from the Crosslink data (i.e. the time and frequency tags  $T_G$ ,  $T_{GA}$ ,  $T_S$ ,  $T_{SA}$ ,  $N_{ED}$ ,  $N_{SU}$ ,  $N_{SD}$ ,  $N_{EU}$ ).

A further source of errors is the accuracy available in estimating the time and frequency tags. This accuracy depends on mainly two factors: (i) the accuracy of the timing recovery procedures and (ii) the on-board oscillator phase noise (the phase noise of the on-ground oscillator is supposed negligible being it based on atomic devices). As far as the first aspect is concerned, adopting the X band for the signal transmission (e.g. 8 GHz), any timing recovery algorithm cannot leave an error less than 1/100 of the carrier period, i.e. 1.25ps. Since it is directly related to  $\delta \approx dT$ , no Doppler frequency between the on-board and on-ground oscillators with precision better than  $1.25 \cdot 10^{-12}$  can be recovered.

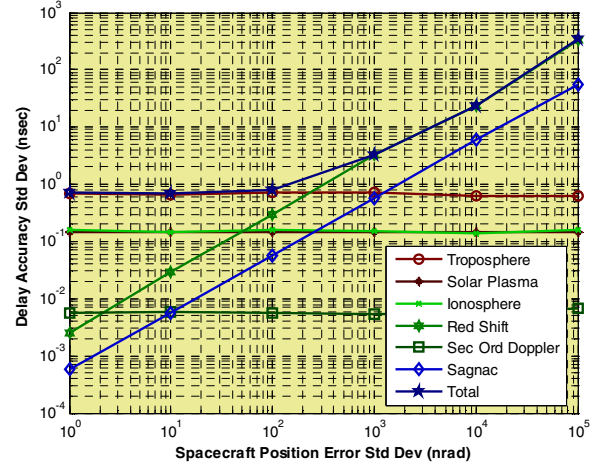


Figure 5. Time Transfer Accuracy

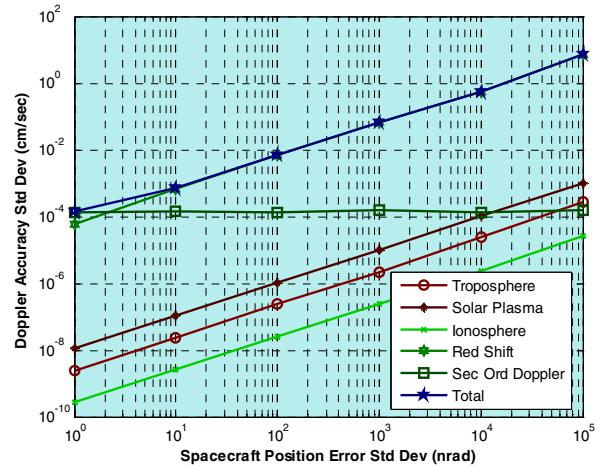


Figure 6. Frequency Transfer Accuracy



As far as the second point is concerned, analysis of the phase noise masks of commercial quartz oscillators provided a standard deviation of their Doppler drift, calculated on the basis of the Jupiter RTT (i.e. about 6000sec), of the same order of magnitude. It is worth noting that this accuracy is proportional to the GS-SC distance and it is reduced in case of nearer missions (e.g. to less than 1m in case of a mission to Mars).

Table II summarises the error budget providing the global accuracy achieved with the analysed technique.

TABLE II. ACCURACY BUDGET

Error Source	Delay STD	Doppler STD
Geometric Asymmetry	6.67ns (2m)	$0.33 \cdot 10^{-12}$ (0.1mm/sec)
Time Estimation Algorithms & Phase Noise	1.25ps (0.375mm)	$1.25 \cdot 10^{-12}$ (0.375mm/sec)
Others Asymmetries	0.80ns (0.24m)	$0.03 \cdot 10^{-12}$ (0.009mm/sec)
<b>Total</b> (separate Delay & Doppler)	6.72ns (2.01m)	$1.29 \cdot 10^{-12}$ (0.387mm/sec)
Impact of Doppler on Delay	3.87ns (1.16m)	
<b>Total</b> (including impact of Doppler on Delay)	7.75ns (2.33m)	

The impact of the Doppler residual error on the Delay estimation accuracy has been obtained through using (9). Still considering in fact a RTT of 6000sec, the total Doppler error  $\epsilon_\delta = 1.29 \cdot 10^{-12}$  shown in Table II can be expressed in terms of delay, range and speed as:

$$\begin{aligned}
\epsilon_{\Delta t} &= \frac{RTT}{2} \cdot \epsilon_\delta = 3.87 \text{ ns} \\
\epsilon_d &= c \cdot \epsilon_{\Delta t} = 1.16 \text{ m} \\
\epsilon_{speed} &= c \cdot \epsilon_\delta = 0.387 \text{ mm/sec}
\end{aligned} \tag{14}$$

#### IV. CONCLUSIONS

A modified version of the classical crosslink technique has been analyzed in this paper for applications concerning deep space scenarios. In particular a three steps procedure has been proposed to achieve (i) the spacecraft Doppler tracking, (ii) the frequency transfer and (iii) the time transfer. The analysis has been carried out taking into account several disturbs which degrade the performances of the technique. A simulator of a specific mission to Jupiter has been developed under MATLAB environment to evaluate the contributions of the involved disturbs. The obtained results, under the constraints of having neither on-board atomic oscillator nor on-board synchronous processing, which are the main responsible of the payload cost as far as the time transfer techniques are

concerned, can be considered of great interest being the achievable time transfer accuracy less than 10 ns.

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